Even Vertex Odd Mean Labeling of Graphs

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KEYWORDS

Even vertex odd mean Labeling, Even vertex odd mean Graph, F-Tree graph, Z-Tree graph, Flag graph

ABSTRACT

A graph with p vertices and q edges is said to have an even vertex odd mean labeling if there exits an injective function $f: V(G) \longrightarrow \{0,2,4, \dots, 2q-2, 2q\}$ such that the induced map $f^*: E(G) \longrightarrow \{1,3,5, \dots 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection.

A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. Here we investigate the even vertex odd mean labeling of $F_{T_n}^{(+)} (n \geq 3)$, Flag graph and other graphs.

1. INTRODUCTION

Throughout this paper, we assign a finite and undirected simple graph. The set of vertices and the set of edges of a graph G is denoted by $V(G)$ and $E(G)$ respectively. A graph labeling is a mapping that carries a set of elements that is vertices and edges.

The concept of mean labeling was introduced and studied by Somasundaram and Ponraj. Further some more results on mean graphs are discussed. A graph G is said to be a mean graph if there is an injective function f from $V(G)$ to $\{0,1,2,3, \dots, q\}$ such that for each edge v , labeled with $\left[\frac{f(u)+f(v)}{2}\right]$ if $f(u)+f(v)$ is even and $\left[\frac{f(u)+f(v)+1}{2}\right]$ if $f(u)+f(v)$ is odd. Then the resulting edge labels are distinct.

K. Manickam and M. Marudai introduced odd mean labeling of a graph. This helps us to define a new concept called Even vertex odd mean labeling of graphs.

A graph G with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function $f : V(G) \longrightarrow \{0, 2, 4, \dots, 2q-2, 2q\}$ such that the induced map $f^* : E(G) \longrightarrow \{1, 3, 5, \dots, 2q-1\}$ defined by $f^*(uv) = [\frac{f(u)+f(v)}{2}]$ is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph.

In this paper we discuss about the path P_n , F-Tree graph, Z-Tree graph and even vertex odd mean labeling of some graphs.

Path on n vertex is denoted by P_n . Fl is called a flag graph. A vertex of degree one is called pendent vertex. An acyclic graph is a graph that contains no cycles. A tree is a connected acyclic graph. The F-Tree graph is a graph obtained by joining two pendent vertex at the end of the path graph. Z-Tree graph is a graph obtained by joining a path to the vertex of two path graphs that is by joining v_n and u_1.

DEFINITION: 1.1

F-Tree $F_T(P_n)$ is a graph obtained from path on $n \geq 3$vertices by appending two pendent edges one to an end vertex and the other to a vertex adjacent to an end vertex.

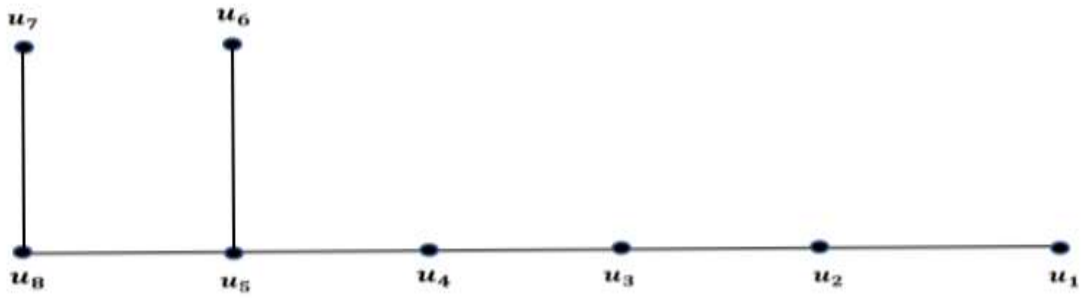

Figure 1.1

DEFINITION: 1.2

A Flag graph Fl is a graph which is obtained from path on $n \geq 3$vertices by appending two pendent edges one to an end vertex and the other to a vertex adjacent to an end vertex and the vertex of the two pendent vertices are adjacent.

DEFINITION 1.3

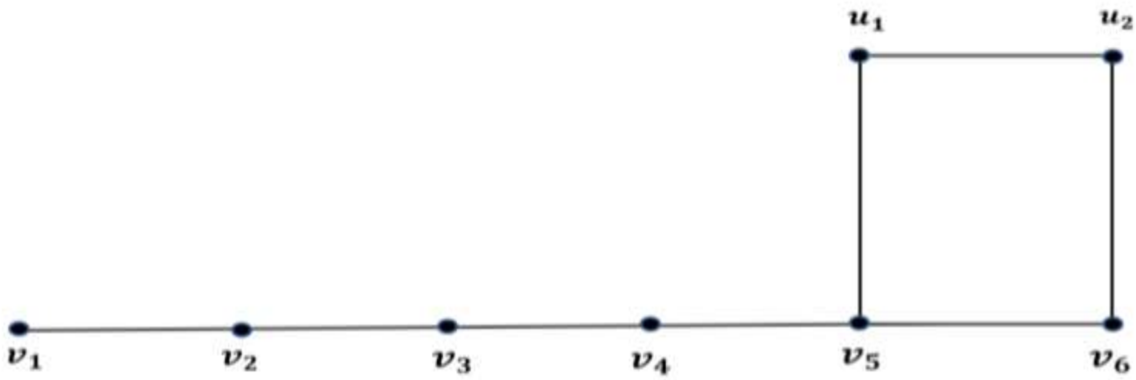

Figure 1.2

Let F_T be a $F -$ Tree graph with $V(G) = \{u_1, u_2, u_3, \dots, u_n\}$ and F_T^* be a copy of F_T with $\{u'_1, u'_2, u'_3, \dots, u'_n\}$. Then the graph $F_{T_n}^{(+)}$ is obtained by joining the vertex u_i with u'_i by an edge of all $1 \leq i \leq n$.

DEFINITION 1.4

The Z-Tree $Z_T(P_n)$ of a path P_n $(n \geq 2)$ is the graph obtained from the copies of P_n with vertices $v_1, v_2, v_3, \dots, v_n$ and $u_1, u_2, u_3, \dots, u_n$ by joining the vertices v_n and u_1.

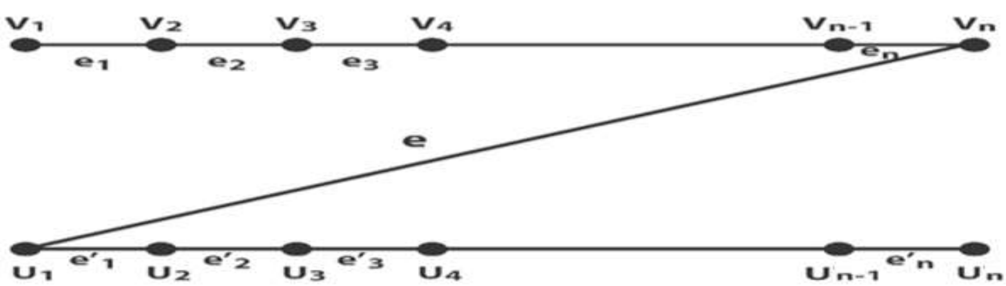

Figure 2.7

DEFINITION 1.5

Let Z_T be a Z–Tree graph with $V(G) = \{ v_1, v_2, v_3, \dots, v_n\}$ and $U(G) = \{ u_1, u_2, u_3, \dots, u_n\}$ and Z_T^* be a copy of Z_T with $v_1', v_2', v_3', \dots, v_n'$ and $u_1', u_2', u_3', \dots, u_n'$ then $Z_{T_n}^{(+)}$ is obtained by joining the vertices of v_i with v_i' and u_i with u_i' by an edge of all $1 \le i \le n$.

2. MAIN RESULTS

THEOREM 2.1

The F_T $(n \ge 3)$ is an even vertex odd mean graph for any n.

PROOF

Let u_i be the vertices and a_i , $1 \le i \le n$ be the edges which are denoted as in Figure 2.1

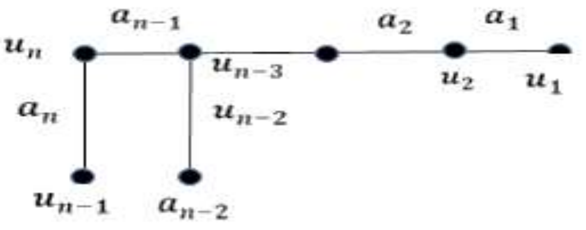

Figure 2.1

First, we label the vertices as follows

Define $f : V \longrightarrow \{0, 2, 4, \dots, 2q\}$ by

For $1 \le i \le n$,

$f(u_i) = 2(i-1)$

Then the induced edge labels are,

For $1 \le i \le n-1$

$f^*(a_i) = 2i - 1$

Therefore, the vertices of labeling of F_T , $1 \le i \le 6$ is defined by

$f(u_1) = 0$
$f(u_2) = 2$
$f(u_3) = 4$
$f(u_4) = 6$
$f(u_5) = 8$
$f(u_6) = 10$

$f^*(a_1) = 1$
$f^*(a_2) = 3$
$f^*(a_3) = 5$
$f^*(a_4) = 7$
$f^*(a_5) = 9$

Therefore $f^*(E) = \{ 1, 3, 5, \dots, 2q-1 \}$. So f is even vertex odd mean labeling and hence the graph F_T $(n \ge 3)$ is an even vertex odd mean graph for any n.

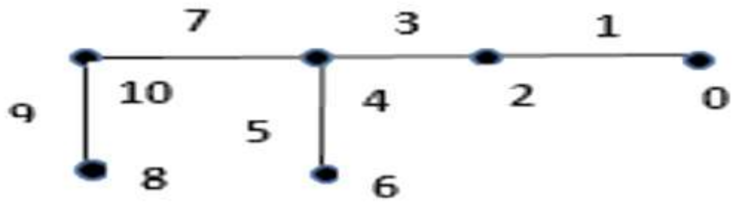

Figure 2.2

THEOREM 2.2

The $F_{T_n}^{(+)} (n \geq 3)$ is an even vertex odd mean graph for any $n \geq 3$.

PROOF

Let $\{ u_i, u_i' \}$ bet the vertices and $\{ a_i, a_i', c_i \}$, $1 \leq i \leq n$ be the edges which are denoted as in Figure 2.3

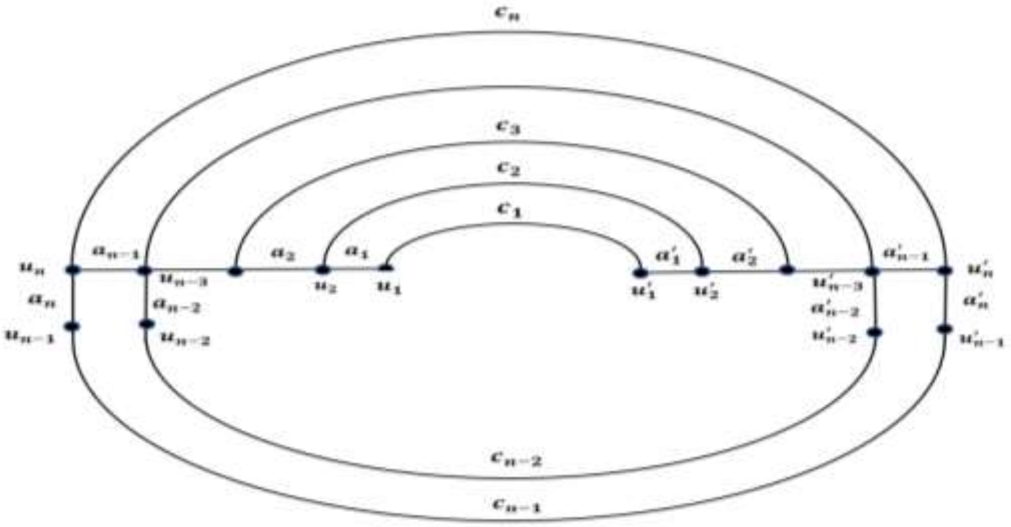

Figure 2.3

First, we label the vertices as follows

Define $f : V \longrightarrow \{0, 2, 4, \dots, 2q\}$ by

For $1 \leq i \leq n$,

$f(u_i) = 2(i-1)$

$f(u_i') = 4n + 2i - 4$

Then the induced edge labels are,

For $1 \leq i \leq n-1$

$f^*(a_i) = 2i - 1$

$f^*(a_i') = 4n + 2i - 3$

$f^*(c_i) = 2n + 2i - 3$

Therefore, the vertices of labeling of $F_{T_n}^{(+)}$, $1 \leq i \leq 6$ is defined by

$f(u_1) = 0$
$f(u_2) = 2$
$f(u_3) = 4$
$f(u_4) = 6$
$f(u_5) = 8$
$f(u_6) = 10$

$f(u_1') = 22$
$f(u_2') = 24$
$f(u_3') = 26$
$f(u_4') = 28$
$f(u_5') = 30$
$f(u_6') = 32$

$f^*(a_1) = 1$

$f^*(a_2) = 3$
$f^*(a_3) = 5$
$f^*(a_4) = 7$
$f^*(a_5) = 9$

$f^*(a_1') = 23$
$f^*(a_2') = 25$
$f^*(a_3') = 27$
$f^*(a_4') = 29$
$f^*(a_5') = 31$

$f^*(c_1) = 11$
$f^*(c_2) = 13$
$f^*(c_3) = 15$
$f^*(c_4) = 17$
$f^*(c_5) = 19$
$f^*(c_6) = 21$
Therefore $f^*(E) = \{ 1, 3, 5, \ldots, 2q-1 \}$. So f is even vertex odd mean labeling and hence the graph $F_{T_n}^{(+)} (n \geq 3)$ is an even vertex odd mean graph for any n.
Even vertex odd mean labeling of $F_{T_6}^{(+)}$ is shown in Figure 2.4

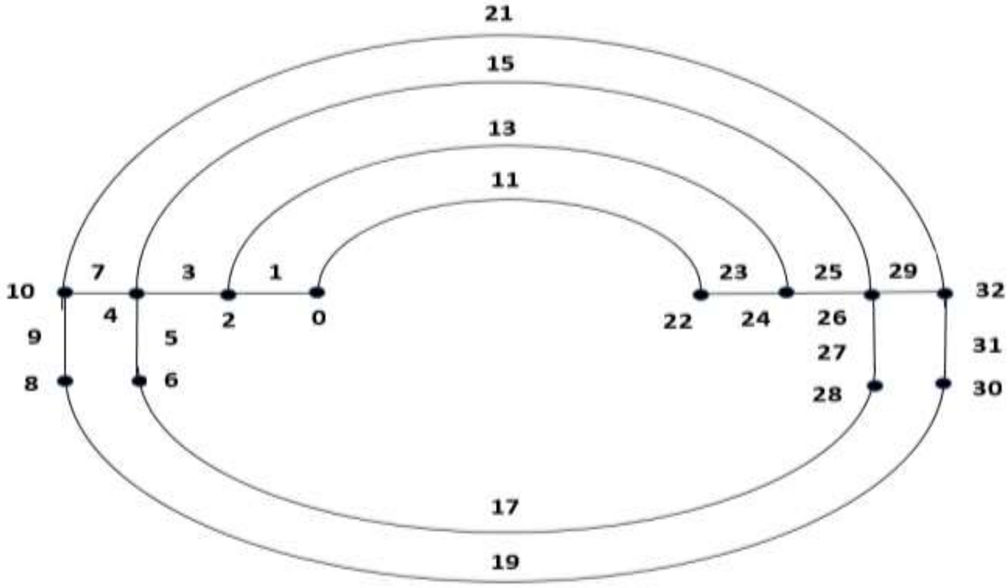

Figure 2.4

THEOREM 2.3
The Flag graph Fl_n is an even vertex odd mean graph for any $n \geq 3$.

PROOF

Let $\{ u_1, u_2, u_3, \dots, u_n\}$ and $\{ v_1, v_2\}$ be the vertices and $\{ e_1, e_2, e, \dots, e_n\}$ be the edges of the graph Fl which are denoted as in Figure 2.5

First, we label the vertices as follows

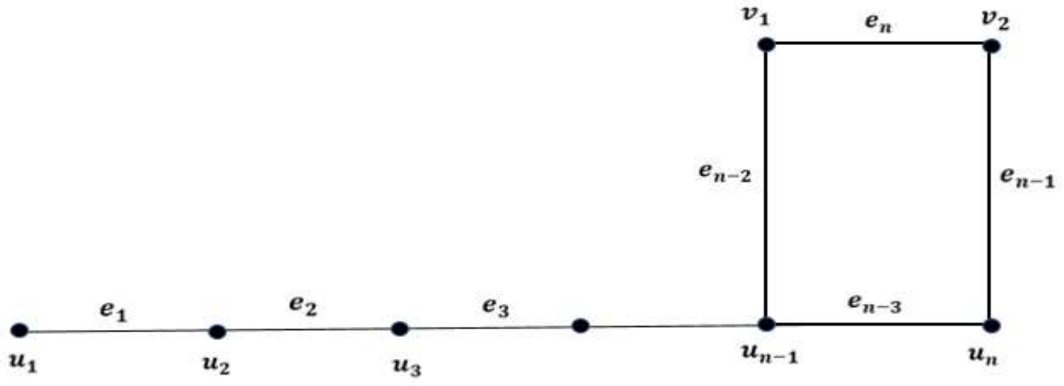

Figure 2.5

Define $f : V \longrightarrow \{0, 2, 4, \dots, 2q\}$ by

$f(u_i) = 2(i-1), 1 \leq i \leq n$

$f(v_i) = 2n + 2i , i = 1,2$

Then the induced edge labels are,

For $1 \leq i \leq n$

$f^*(e_i) = 2i - 1$

Therefore $f^*(E) = \{ 1, 3, 5, \dots, 2q - 1 \}$. So f is even vertex odd mean labeling and hence the graph Fl_n is an even vertex odd mean graph for any n.

Therefore, the vertices of labeling of Fl_n, $1 \leq i \leq 6$ is defined by

$f(u_1) = 0$
$f(u_2) = 2$
$f(u_3) = 4$
$f(u_4) = 6$
$f(u_5) = 8$
$f(u_6) = 10$

$f(v_1) = 14$
$f(v_2) = 16$

$f^*(e_1) = 1$
$f^*(e_2) = 3$
$f^*(e_3) = 5$
$f^*(e_4) = 7$
$f^*(e_5) = 9$
$f^*(e_6) = 11$

Even vertex odd mean labeling of Fl_6 is shown in Figure 2.6

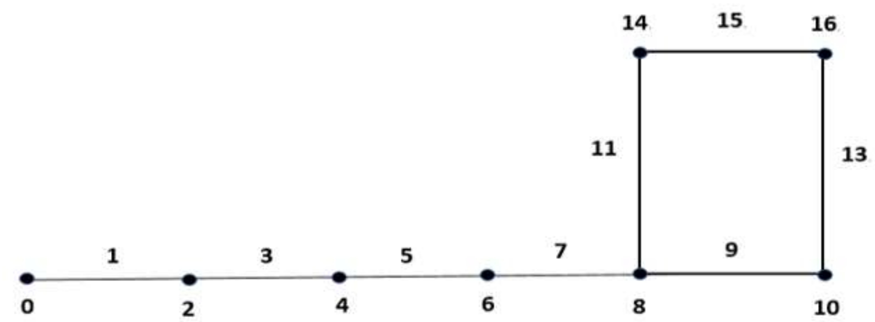

Figure 2.6

THEOREM 2.4
The Z-Tree graph Z_T is an even vertex odd mean graph for any $n \geq 2$.
PROOF
Let $\{v_1, v_2, v_3, \dots, v_n\}$ and $\{u_1, u_2, u_3, \dots, u_n\}$ be the vertices and $\{e_1, e_2, e, \dots, e_n\}$, $\{e_1', e_2', e_3', \dots, e_n'\}$ and $\{ e \}$ be the edges of the graph Z-Tree which are denoted as in Figure 2.7

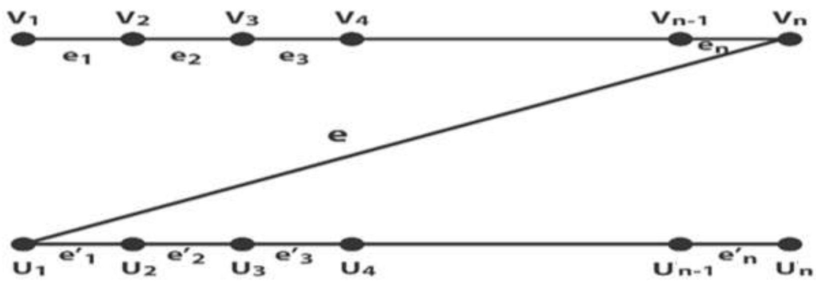

Figure 2.7

We now assign a vertices as follows
Define a function $f : V \longrightarrow \{0, 2, 4, \dots, 2q\}$ by
$f(v_i) = 2(i-1), \qquad 1 \leq i \leq n$
$f(u_i) = 2n + 2(i-1), \qquad 1 \leq i \leq n$
The induced edge labels are,
$f^*(e_i) = 2i - 1$
$f^*(e_i') = 2n + 2i - 1$
$f^*(e) = 2n - 1$
Therefore, $f^*(E) = \{1, 3, 5, \dots, 2q - 1\}$
So f is even vertex odd mean labeling and hence the graph $Z_T(P_n)$ is an even vertex odd mean graph for any $n \geq 2$.
Therefore, the vertices of labeling of Z_T, $1 \leq i \leq 5$ is defined by

$f(v_1) = 0$
$f(v_2) = 2$
$f(v_3) = 4$
$f(v_4) = 6$
$f(v_5) = 8$

$f(u_1) = 10$
$f(u_2) = 12$
$f(u_3) = 14$

$f(u_4) = 16$
$f(u_5) = 18$

$f^*(e_1) = 1$
$f^*(e_2) = 3$
$f^*(e_3) = 5$
$f^*(e_4) = 7$

$f^*(e) = 9$

$f^*(e'_1) = 11$
$f^*(e'_2) = 13$
$f^*(e'_3) = 15$
$f^*(e'_4) = 17$

Even vertex odd mean labeling of $Z_T(P_5)$ is shown in figure 2.8

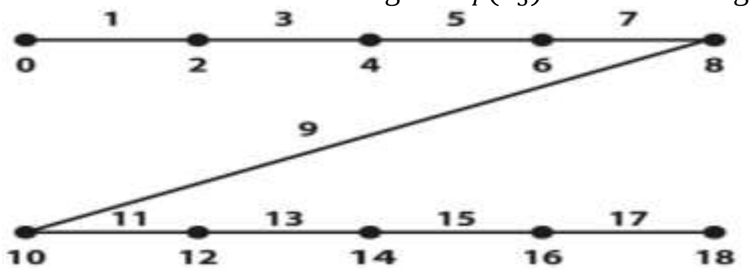

Figure 2.8

THEOREM 2.5

The graph $Z_{T_n}^{(+)}$, $n \geq 2$ is an even vertex odd mean graph for any n.

PROOF

Let $\{ v_i, \ v'_i, u_i, \ u'_{i,}\}$ be the vertices and $\{ \ a_i, \ a'_i, b_i, \ b'_{i,} e_i, \ e'_i, \ c_i \ \}$, $1 \leq i \leq n$ be the edges which are denoted as shown in the figure 2.9

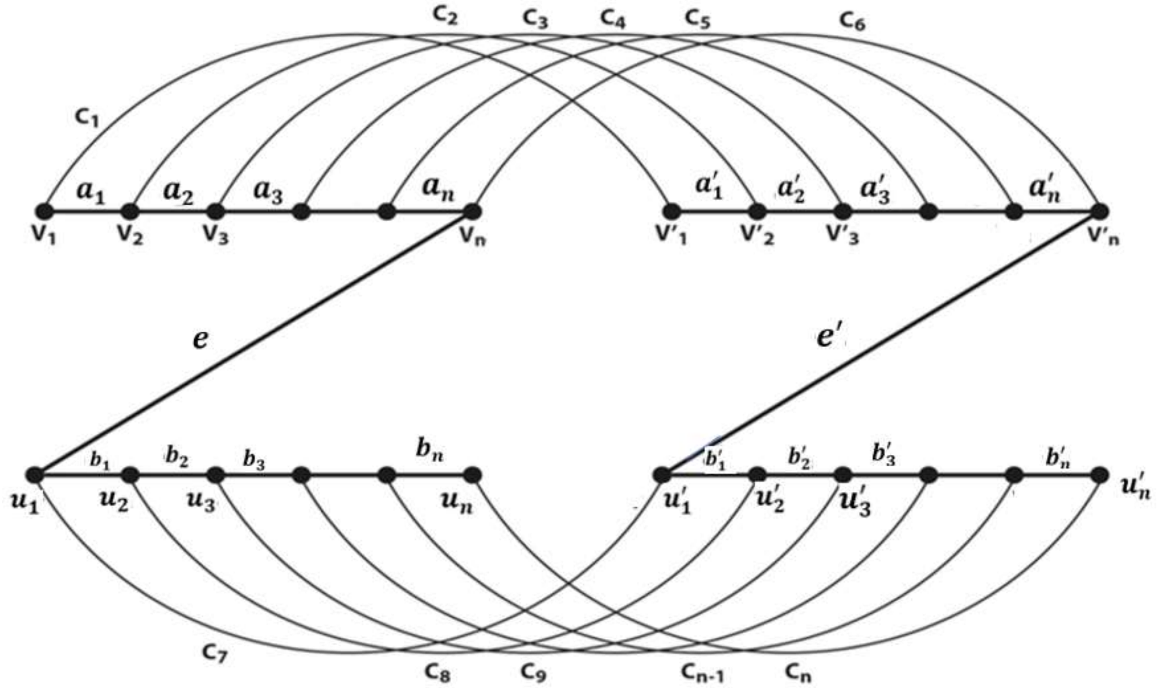

Figure 2.9

First we label the vertices by defining the function, $f : V \longrightarrow \{0, 2, 4, \dots, 2q\}$

For $1 \leq i \leq n$,
$f(v_i) = 2(i-1)$
$f(u_i) = 2n + 2(i-1)$
$f(v_i') = 8n + 2(i-2)$
$f(u_i') = 10n + 2(i-2)$
Then the induced edge labels are,
$f^*(a_i) = 2i - 1$
$f^*(b_i) = 2n + 2i - 1$
$f^*(e) = 2n - 1$
$f^*(a_i') = 8n + 2i - 3$
$f^*(b_i') = 10n + 2i - 3$
$f^*(e') = 10n - 3$
$f^*(c_i) = 4n + 2i - 3$
Therefore, the vertices of labeling of $\boldsymbol{Z}_{\boldsymbol{T_3}}^{(+)}$ is defined by,
$f(v_1) = 0$
$f(v_2) = 2$
$f(v_3) = 4$
$f(u_1) = 6$
$f(u_2) = 8$
$f(u_3) = 10$

$f(v_1') = 22$
$f(v_2') = 24$
$f(v_3') = 26$
$f(u_1') = 28$
$f(u_2') = 30$
$f(u_3') = 32$
$f^*(a_1) = 1$
$f^*(a_2) = 3$
$f^*(e) = 5$
$f^*(b_1) = 7$
$f^*(b_2) = 9$
$f^*(a_1') = 23$
$f^*(a_2') = 25$
$f^*(e') = 27$
$f^*(b_1') = 29$
$f^*(b_2') = 31$
$f^*(c_1) = 11$
$f^*(c_2) = 13$
$f^*(c_3) = 15$
$f^*(c_4) = 17$
$f^*(c_5) = 19$
$f^*(c_6) = 21$
Therefore$f^*(E) = \{ 1, 3, 5, \dots, 2q - 1 \}$. So f is even vertex odd mean labeling and hence the graph $Z_{T_n}^{(+)}(n \geq 2)$ is an even vertex odd mean graph for any n.
Even vertex odd mean labeling of $\boldsymbol{Z}_{\boldsymbol{T_3}}^{(+)}$ is shown in Figure 2.10

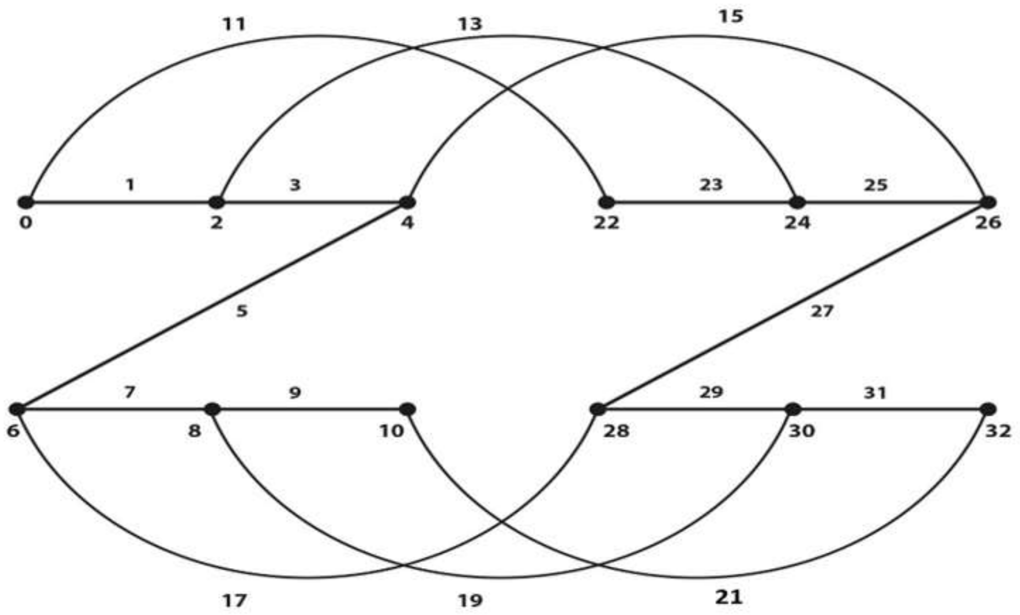

Figure 2.10

CONCLUSION:

Here we proved the graphs F-tree, $F_{T_n}^{(+)}$ graph, Flag graph (Fl_n) , Z-Tree and $Z_{T_n}^{(+)}$ graph admits even vertex odd mean labeling. As all graphs are not even vertex odd mean graph and it is interesting to investigate similar results for other graphs.

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